Cost of Not Arbitrarily Splitting in Routing

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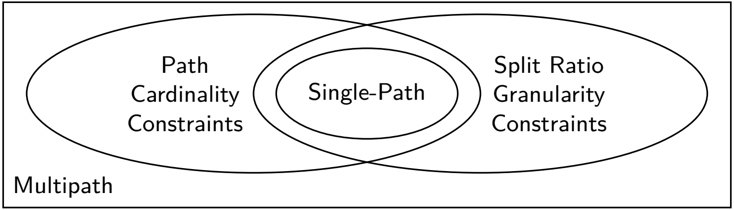
Outline

Introduction

- Practical restrictions can prevent an optimized routing solution to be fully realized.
- Routers can put additional restrictions on routing solutions such as:
  - At most W paths are allowed for each source-destination pair (Path cardinality constraints).
  - There is a minimum granularity for the split ratio (Split ratio granularity constraints).



Introduction

- Multipath routing achieves the best performance, but is hard to implement. Single-path routing is opposite.
- When the split ratio granularity becomes larger, routing performance decreases but implementation overhead also decreases.

Two Basic Questions

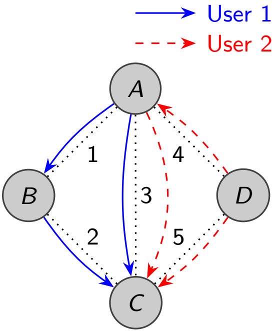
- How to estimate the performance gap between the multipath routing and routing with split ratio granularity constraints?
- How to find a good approximate solution to the split ratio granularity problem?

Notation

- Number of links $L = 5$. Number of users $N = 2$. Number of paths each user has $K^1 = K^2 = 2$.
- The paths of user i are represented by

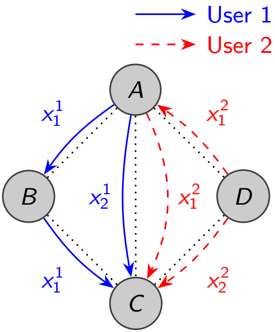
$$R^1 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad R^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Put $R^i_{lk} = 1$ if path k of user i passes through link l.



Notation

- Link capacity $c = (c_1, \ldots, c_5)^T$.
- The kth entry x_k^i of vector x^i is the sending rate of user i on path k.
- The utility $U^i(\cdot)$ of user i is a function of its total transmission rate $\|x^i\|_1$.
- We focus on two common utility functions:
  - Linear utility $U^i(s) = s$.
  - Logarithmic utility $U^i(s) = \log s$.



Multipath Routing

The network utility maximization (NUM) problem with multipath routing:

$$\begin{aligned} \max \quad & \sum_{i=1}^{N} U^i\left(\|x^i\|_1\right) \\ \text{s.t.} \quad & \sum_{i=1}^{N} R^i x^i \leq c, \\ & x^i \in \mathcal{I}_{K^i}, \quad \forall i = 1, \ldots, N. \end{aligned}$$

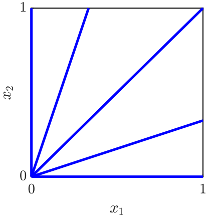
- Here $\mathcal{I}_K = \{x \in \mathbb{R}^K | 0 \leq x_k \leq \|c\|_\infty, k = 1, \ldots, K\}$, while $\|c\|_\infty$ is the maximum capacity among all links in the network.
- Without loss of generality, assume $\|c\|_\infty = 1$.
- Replace $\mathcal{I}_K$ by a smaller set to introduce split ratio granularity constraints.

Split Ratio Granularity Constraints

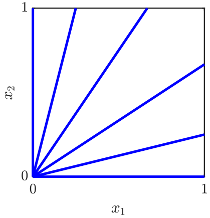
- Suppose the split ratio of each user needs to be a multiple of $1/p$, where p is a given integer.
- Choose the set

$$\mathcal{S}_K = \{0\} \cup \left\{ x \in \mathcal{I}_K \middle| x \neq 0, \frac{p x_k}{\|x\|_1} \in \mathbb{Z}, k = 1, \ldots, K \right\}$$

to replace $\mathcal{I}_K$ in the NUM problem.



$p = 4$



$p = 5$

Outline

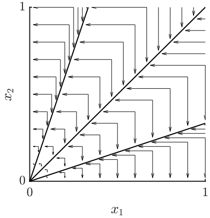
Bounding the Performance Gap

- There exists an optimal solution to the multipath problem sending positive rates on at most $N + L$ paths.
- For the linear utility case, the performance gap is bounded by

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{N} \rho_{\tilde{K}^i} \\
\text{s.t.} \quad & \sum_{i=1}^{N} \tilde{K}^i \leq N + L, \\
& 0 \leq \tilde{K}^i \leq K^i,\ \tilde{K}^i \in \mathbb{Z}, \quad \forall i = 1, \dots, N.
\end{aligned}
$$

Optimal Rounding

- For a rate vector x, the optimal rounding y of x is a rate vector maximizing the total transmission rate such that the split ratio granularity constraints are satisfied and $y \leq x$.



- ρ_K is the maximum throughput loss during the rounding for a user using K paths.
- For the logarithmic utility case, ρ_K is replaced by $\log \rho_K^R$, which is the maximum relative throughput loss during rounding.

$$\rho_K = \max_{\Gamma = p, \ldots, p+K-1} \frac{\Gamma - p}{\lceil \Gamma / K \rceil}, \quad \rho_K^R = \frac{K-1}{p+K-1}.$$

Outline

Problem Formulation

Bounding the Performance Gap

Routing Optimization with Split Ratio Granularity Constraints

Solving the Split Ratio Granularity Problem

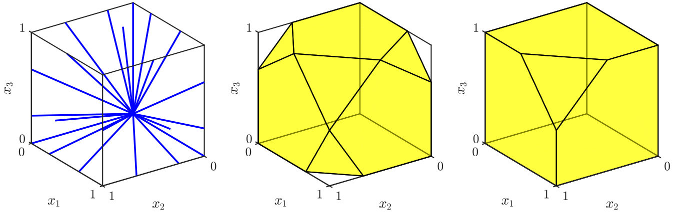
- NP-hard to find the optimal solution to the split ratio granularity problem.
- The approach mentioned above provides an approximate solution based on the multipath problem.
- Can find a tighter relaxation that leads to a better performance guarantee.

Convex Relaxation

Replace the constraint set $\mathcal{S}_K$ in the split ratio granularity problem by a convex set $\mathcal{T}_K$ satisfying $\mathcal{S}_K \subseteq \mathcal{T}_K \subseteq \mathcal{I}_K$:

$$\mathcal{T}_K = \{x \in \mathcal{I}_K | \|x\|_1 \leq C_K\}.$$

Here C_K is the maximum throughput over all rate vectors in $\mathcal{S}_K$.

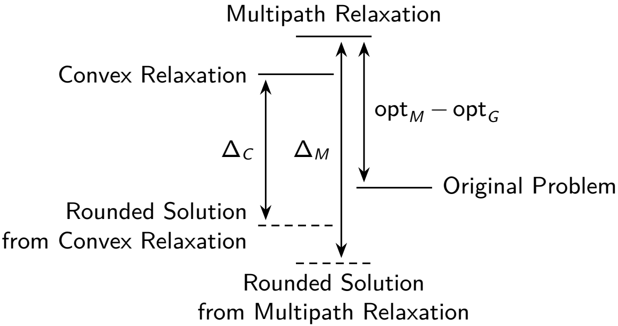


Convex Relaxation

- The performance guarantee for the approximation algorithm using convex relaxation is determined by ρ_K^C, which is the maximum throughput loss during the rounding for a rate vector in $\mathcal{T}_K$.
- ρ_K^C is either equal to ρ_K or

$$\frac{K-1}{p+K-1}C_K.$$

Summary


Multipath Relaxation
Convex Relaxation
$opt_M - opt_G$
Δ_C
Δ_M
Original Problem
Rounded Solution
from Convex Relaxation
Rounded Solution
from Multipath Relaxation

Thank You!