Uncertainty-Aware Optimization for Network Provisioning and Routing

Yingjie Bi

School of Electrical and Computer Engineering
Cornell University

Joint work with Kevin Tang

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Outline

Introduction

Two main goals of traffic engineering:

- Accommodate the demands from customers.
- Minimize the network cost.

Previous approaches to handle traffic fluctuations in traffic engineering:

- Oblivious routing.
- Dynamic routing.

Our Approach

- Combines the ideas from both oblivious routing and dynamic routing.
- Utilizes the different flexibility of network reconfiguration and characteristics of traffic demand in different timescales.
- Applies to wide-area backbone network.

Our Approach

Long-term timescale:

- Typically within a few months.
- Operators can freely reconfigure the network.
- Very limited information on the demand side.

Short-term timescale:

- Approximately from an hour to several minutes.
- Operators cannot change the network topology but can adjust the routing.
- The traffic demand is more predictable.

Transient timescale:

- Seconds or below.
- Operators cannot perform any change to the network.
- Demand information is most complete.

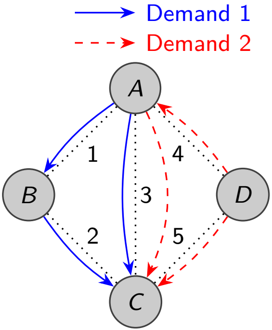
Outline

Notation

- Number of links $L = 5$. Number of demands $N = 2$. Number of available paths for each demand $K^1 = K^2 = 2$.
- The paths of demand i are represented by

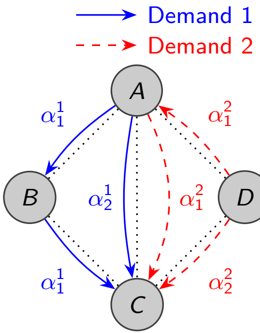
$$R^1 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad R^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Put $R^i_{lk} = 1$ if path k of demand i passes through link l.



Notation

- Link capacity $c = (c_1, \ldots, c_5)^T$.
- The kth entry α^i_k of vector α^i is the split ratio of demand i on path k.



Traffic Model

We adopt the following traffic model (Tune and Roughan, 2013):
The traffic volume of demand i

$$D^i(t) = L^i(t)S^i(t) + \sqrt{\beta^i L^i(t)S^i(t)}W^i(t),$$

where

- $L^i(t)$ is the long-term trend.
- $S^i(t)$ is the seasonal component.
- $W^i(t)$ is the random fluctuations.
- β^i is a constant called the peakedness of the traffic.

Traffic Model

Further assumptions on the traffic model:

- Approximate the seasonal component $S^i(t)$ by Q scenarios, i.e.,
$$S^i(t) = S^i_{q(t)},$$
where $q(t)$ is a periodic and piecewise constant function whose value is an integer between 1 and Q.
- $W^i(t)$ is assumed to be a white Gaussian process with zero mean and unit variance.

Summary

Characteristics of the three timescales in network optimization:

Timescale	Typical Granularity	Known Information	Decision Variables
Long-term	A month	$L^i(t)$	c_l, α_k^i
Short-term	An hour	$L^i(t)$, $S^i(t)$	α_k^i
Transient	A second	$L^i(t)$, $S^i(t)$, $W^i(t)$	None

Outline

Short-Term Optimization

- The actual traffic Y_l on link l

$$Y_l = \sum_{i=1}^{N} \sum_{k=1}^{K^i} R^i_{lk} \alpha^i_k D^i.$$

- The goal is to minimize the probability of link capacity violations, i.e.,

$$\begin{aligned}
\min \quad & \Pr\left(\bigcup_{l=1}^{L} \{ Y_l > c_l \} \right) \\
\text{s. t.} \quad & \sum_{k=1}^{K^i} \alpha^i_k = 1, \quad \forall i = 1, \ldots, N, \\
& \alpha^i_k \geq 0, \quad \forall i = 1, \ldots, N,\ k = 1, \ldots, K^i.
\end{aligned}$$

Long-Term Optimization

- Assume p_l is the cost of link l per unit capacity.
- The goal is to minimize the aggregate cost while the violation probability is not exceeding ϵ, i.e.,

$$\begin{aligned}
\min \quad & \sum_{l=1}^{L} p_l c_l \\
\text{s.t.} \quad & \Pr\left(\bigcup_{l=1}^{L} \{ Y_{l,q} > c_l \} \right) \leq \epsilon, \quad \forall q = 1, \ldots, Q, \\
& \sum_{k=1}^{K^i} \alpha_{k,q}^i = 1, \quad \forall i = 1, \ldots, N,\ q = 1, \ldots, Q, \\
& \alpha_{k,q}^i \geq 0, \quad \forall i = 1, \ldots, N,\ k = 1, \ldots, K^i, \\
& \qquad q = 1, \ldots, Q,
\end{aligned}$$

where $Y_{l,q}$ is the actual traffic on link l at scenario q.

Approximate Solutions

- Both the short-term and long-term optimization problems are difficult to solve.
- For the long-term problem, constraint the violation probability of each link separately by setting

$$\Pr(Y_{l,q} > c_l) \le \epsilon/L, \quad l = 1, \ldots, L.$$

  instead of the original probability constraints.
- $Y_{l,q}$ is subject to Gaussian distribution whose mean $y_{l,q}$ and variance $b_{l,q}$ can be obtained from the split ratio $\alpha^i_{k,q}$ and the traffic model.

Approximate Solutions

- Let $f(x)$ be the survival function of standard normal distribution, i.e.,
$$f(x) = \Pr(X > x),$$
where X is subject to the standard normal distribution.
- Define
$$A = \frac{1}{f^{-1}(\epsilon/L)}.$$
The above constraint
$$\Pr(Y_{l,q} > c_l) \le \epsilon/L$$
is equivalent to
$$\frac{c_l - y_{l,q}}{\sqrt{b_{l,q}}} \ge \frac{1}{A}.$$
- This is a second-order cone constraint which can be handled efficiently by convex optimization techniques.

Performance Guarantees

Let

- sol be the optimal value for the approximate problem and
- opt be the actual optimal cost.

Then the approximation ratio

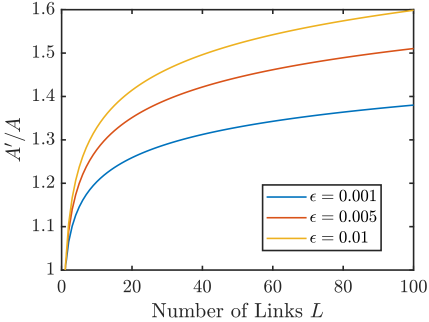
$$\text{sol} / \text{opt} \leq A'/A = O(\sqrt{\log L / \log(1/\epsilon)}),$$

where

$$A = \frac{1}{f^{-1}(\epsilon/L)}, \quad A' = \frac{1}{f^{-1}(\epsilon)}.$$

Performance Guarantees

The actual dependence of the approximation ratio sol / opt on the number of links L and the violation probability ϵ:



Thank You!